line sections of impedance $Z_{12}=Z_{23}=R_B$ have been added on the right to allow the use of Kuroda's identity in the next step. Since their characteristic impedance matches that of the termination, they do not affect the attenuation characteristic of the circuit; their only effect on the response is to give some added phase shift. The circuit in Fig. 19 b) then has a response which is the desired exact mapping of the low-pass prototype response. The only trouble with the filter in Fig. 19 b) is that it contains a series stub which is difficult to construct in a shielded TEM-mode microwave structure.

The series stub in Fig. 19 b) can be eliminated by application of Kuroda's identity (Fig. 18). Applying Kuroda's identity to stub Y_3 and line Z_{12} in Fig. 19 b) gives the circuit in Fig. 19 c). Then applying Kuroda's identity simultaneously to stub Z_2 and line Z_{12}' and stub Z_3' and line Z_{23} in Fig. 19 c) gives the circuit in Fig. 19 d). Note that the circuit in Fig. 19 d) has exactly the same input impedance and over-all transmission properties as the circuit in Fig. 19 b), while the circuit in Fig. 19 d) has no series stubs.

The equations in Tables II and III were derived by use of repeated applications of the procedures described above. For reasons of convenience, the equations in the tables use a somewhat different notation than does the example in Fig. 19; however, the principles used are the same. The equations in Tables II and III also provide for a shift in impedance level from that of the low-pass prototype.

ACKNOWLEDGMENT

R. B. Larrick and P. R. Reznek very ably performed the measurements and adjustments on the experimental filters. The assistance of Dr. Leo Young and P. H. Omlor in adapting an existing program for computing the filter responses is most gratefully acknowledged.

A New Microwave Measurement Technique to Characterize Diodes and an 800-Gc Cutoff Frequency Varactor at Zero Volts Bias*

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Summary—A means has been found which enables one to make negligible the complicating effects of posts and shunting cartridge capacitance usually present in microwave diode circuits. This simplification permits the representation of the diode by a simple equivalent circuit and the determination of the effective diode parameters from transmission measurements. Parameters so obtained at X band (8.2 to 12.4 Gc) are compared with audio frequency bridge measurements. Measurements at M band (50–60 Gc) of a new 800-Gc cutoff frequency varactor are also described. This varactor has zero bias junction capacitance in the 0.016 pf range and spreading resistance on the order of 12 Ω. It is expected to extend the useful range of parametric devices well into the millimeter region.

I. INTRODUCTION

THE CHARACTERISTICS of microwave diodes are usually difficult to determine at microwave frequencies. This is primarily due to so called "parasitics" arising from contacting elements, shunting cartridge capacity and supportive posts.[1] These can be "calibrated" out in the standard[2] reflection coefficient techniques, but at high microwave frequencies where circuits become fairly lossy this method meets with considerable difficulty. This calibration can also be a strong function of temperature, which causes difficulties when one is trying to evaluate diode characteristics over a wide temperature range. In addition, this technique has the unfortunate characteristic of decreasing accuracy of measurement of diode characteristics at a given frequency, with increasing diode quality factor.

In a search for effective equivalent circuit parameters for parametric amplifiers, a technique has been developed for characterizing microwave diodes which does not have the disadvantages listed above. This technique employs the series resonance of a diode in reduced height rectangular waveguide in the form presented here, but is adaptable to other types of transmission lines such as

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† Bell Telephone Laboratories, Inc. Murray Hill, N. J.

[1] B. C. DeLoach, "Waveguide parametric amplifiers," *Digest of Tech. Papers of the Internatl. Solid-State Circuits Conf.*, pp. 24–25, 1961.

[2] M. C. Waltz, "A technique for the measurement of microwave impedance in the junction region of a semiconductor device," *Microwave J.*, vol. 2, pp. 23–27; May, 1959.
N. Houlding, "Measurement of varactor quality," *Microwave J.*, vol. 3, pp. 40–45; January, 1960.
R. T. Harrison, "Parametric diode Q measurement," *Microwave J.*, vol. 3, pp. 43–46; May, 1960.

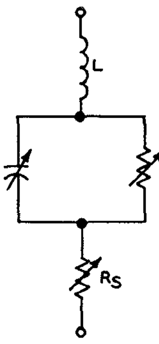

Fig. 1—Diode equivalent circuit with arrows indicating possible voltage dependence of associated symbols.

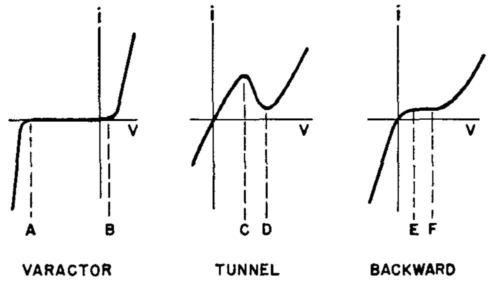

Fig. 2—Typical i, v curves for varactor, tunnel and backward diodes.

coaxial lines[3] and strip lines. It requires only relative power measurements which are much simpler to perform than the phase and magnitude measurements of the conventional reflection technique. Its accuracy is believed to exceed that of any other known method at high microwave frequencies.

The reduced height waveguide technique has now been used to characterize varactor, tunnel and backward diodes. It has been employed over a frequency range from 1 to 60 Gc with varying types of diode cartridges. It is, to our knowledge, the only method presently capable of accurately measuring at *M* band (50–75 Gc) the high quality factor diodes described in this paper whose junction capacities are on the order of 0.01 pf, *i.e.*, 1×10^{-14} f.

II. Microwave Diode Equivalent Circuits

It is beyond the scope of this paper to develop the appropriate solid-state theory to predict the equivalent circuits for different types of microwave diodes. However, the circuit of Fig. 1 has been shown to be sufficient (neglecting parasitics other than the series inductance) to characterize a large class of microwave diodes, and thus we will consider it in some detail with regard to the typical i-v curves of Fig. 2.

The inductance in Fig. 1 is assumed to be primarily due to leads attached to the material containing the junction, and as such is a function of the circuit in which the diode is to be placed, but is little affected by the type of diode involved. However, C the junction capacity, R the barrier resistance (which can be positive or negative) and R_s the series resistance are principally junction properties which vary quite widely depending upon the type of junction employed. In a practical circuit the effective series resistance also includes circuit losses exterior to the junction itself. C, R and R_s can in general be functions of voltage.

Consider the curves of Fig. 2. For the varactor for voltages between A and B, R is quite high (usually many megohms at low frequencies). At voltages C and D for the tunnel and E and F for the backward diode, the ac small signal resistance is quite large and becomes infinite for vanishingly small signals. Then for these bias voltages we can assume $R \gg 1/\omega c$, and the microwave ac small signal equivalent circuit of Fig. 1 may be reduced to that of a series L, C, R_s combination. If we could place this equivalent circuit across a transmission line as is indicated in Fig. 3, with no complicating parasitics, we could then determine L, C, and R_s from the transmission characteristic in a straightforward manner.

III. Theory

We consider the properties of the circuit depicted in Fig. 3, in which a series L, C, R_s combination is shown connected across a transmission line of characteristic impedance Z_0 (for the present assumed independent of frequency) which has both ends match terminated. The transmission response of the L, C, R_s combination is a simple resonance curve with a minimum in the transmission occurring at resonance. A convenient set of measurements on the transmission characteristic includes:

1) The power transmission loss ratio T at resonance,
2) The two frequencies f_1 and f_2 at which the power transmitted to the load resistance is twice that at resonance.

One can then show that

$$R_s = \frac{Z_0}{2} \frac{1}{\sqrt{T} - 1}, \tag{1}$$

$$C = \frac{1}{\pi Z_0} \frac{f_1 - f_2}{f_1 f_2} (\sqrt{T} - 1) \left(1 - \frac{2}{T}\right)^{1/2}, \tag{2}$$

$$L = \frac{1}{4\pi^2} \frac{1}{f_1 f_2 C} \cdot \tag{3}$$

(See Appendix I.) Thus, knowing Z_0, the measurement of T determines R_s and the measurement of T, f_1, and f_2, determines both C and L. It is to be noted that these are simple power and frequency measurements with no necessity for phase determination.

If the transmission line is a dispersive medium, as is rectangular waveguide, such that the characteristic impedance of the transmission line is a function of frequency, it may be necessary to go to a more exact

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[3] A. E. Bakanowski, "Discussion of a Transmission Method for Small Signal Characterization of Varactor Diodes," Bell Telephone Labs., Fifth Interim Rept. on Microwave Solid-State Devices, Signal Corps Contract No. DA 36-039 SC 85325, pp. 30–44; August, 1961.

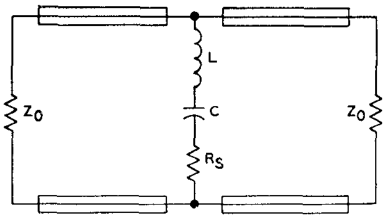

Fig. 3—Match terminated transmission line with a series L, C, R_s combination connected across it.

formulation including this frequency dependance. (See Appendix II.) For relatively high Q diodes this is generally not necessary, even in rectangular waveguide, unless extreme accuracy is required.

IV. Properties of Diodes in Rectangular Waveguides

In attempting to apply the theory of Section III to a diode in ordinary rectangular waveguide several difficulties arise. Rectangular waveguides operated in the TE_{10} dominant mode have the interesting property that several different impedances may be attributed to them. Thus the question of the appropriate Z_0 to use in (1), (2) and (3) must be considered. Fortunately Schelkunoff[4] has shown that the impedance seen by a small diameter wire connected between the centers of the broad walls of a reduced height (height small compare with a quarter wavelength) rectangular waveguide is given by

$$Z_0 = 754 \frac{b}{a} \frac{1}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} \tag{4}$$

for the standard operating range (dominant mode propagation only) of the waveguide.

Additional complications arise if the diode is mounted in a removable cartridge. Cartridges in general contribute inductance and capacitance that can transform the simple circuit of Fig. 3 into something much more complicated. Each such package must be considered separately and one common type will be discussed in Section V.

V. Experimental Measurement of Varactor Diodes at X-Band

X-band measurements are particularly appealing for establishing the validity of our technique due to the availability of excellent measuring equipment, convenient physical dimensions and the fact that junction capacities, that can be series resonated in reduced height X-band waveguide (0.900 in × 0.050 in) are still large enough that they can be measured with reasonable precision on an ordinary capacitance bridge. Measurements in the reduced height waveguide can thus be compared with other measurement procedures on the same diode. This comparison is almost impossible when the diode is "built into" a waveguide structure. This measurement difficulty enhances the value of the transmission technique at high microwave frequencies where waveguide fabrication is employed almost exclusively.

Our experimental arrangement for the measurement of diodes at X band is shown in Fig. 4. The tapered height waveguide sections are electroformed raised cosine tapers,[5] each with a return loss greater than 36 db, from 8.2 to 12.4 Gc. The reduced height section is 0.900 in × 0.050 in and from (4) we obtain the appropriate Z_0 for a given frequency as presented in Fig. 5.

The diodes that we will discuss in this section were mounted in cartridges of the type depicted in Fig. 6 developed by Sharpless.[6] These cartridges are inserted through holes in the centers of the broad walls of the reduced height waveguide section as shown in Fig. 7. The inner faces of the end caps are observed to be flush with the inner walls of the waveguide. The shunting effect of the cartridge capacity is thus considerably reduced, since all that we need concern ourselves with is excess capacitance over and above the distributed capacitance of the waveguide itself. The short lengths of coaxial line extending up into the end caps aid in this respect, in that they tend to decrease the excess capacity introduced by the quartz (while unavoidably adding inductance in series with the junction capacity). Thus it would not be too surprising to find that a simple series L, C, R_s circuit is a good representation for a diode mounted in this cartridge (which has $R \gg 1/\omega c$) at least in the vicinity of the series resonance, when the cartridge is mounted in reduced height (0.050 in high) waveguide and provided that R_s is considerably smaller than the effective shunting cartridge reactance.

Fig. 8 contains experimentally obtained transmission loss curves for a typical reverse biased varactor diode, mounted as previously discussed. The microwave power level at the diode was kept below one microwatt to minimize averaging effects. To assure the absence of these effects the power level was increased 6 db and the readings repeated. With the T, f_1 and f_2 values obtained from each of these curves, one can obtain corresponding values of R_s, C and L either from (1), (2) and (3) or from the more exact formulation in Appendix II, assuming that our equivalent circuit is valid. The value of Z_0 at the resonant frequency $f_0 = \sqrt{f_1 f_2}$ as obtained from Fig. 5 is employed in the approximate calculations. The values obtained from (1), (2) and (3) are presented in Figs. 9 and 10. Included in Fig. 9 is a plot of the junction capacity (the empty cartridge capacity of $0.4 \times 10^{-13} f$ has been subtracted from each point) as

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[4] S. A. Schelkunoff, "Electromagnetic waves," D. Van Nostrand Company, Inc., Princeton, N. J., pp. 494–495; 1943. It should be noted that there are several misprints in the equations for R and X on page 495 as related by S. A. Schelkunoff in a private communication.

[5] F. Bolinder, "Fourier Transforms in the Theory of Inhomogeneous Transmission Lines," Trans. of the Royal Inst. of Technology Stockholm, Sweden, no. 48; 1951.

[6] W. M. Sharpless, "Gallium arsenide point contact diodes," IRE Trans. on Microwave Theory and Techniques, vol. 9, pp. 6–10; January, 1961.

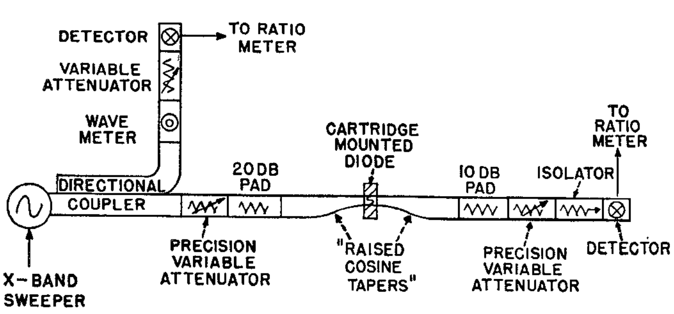

Fig. 4—Experimental arrangement for X-band diode measurements.

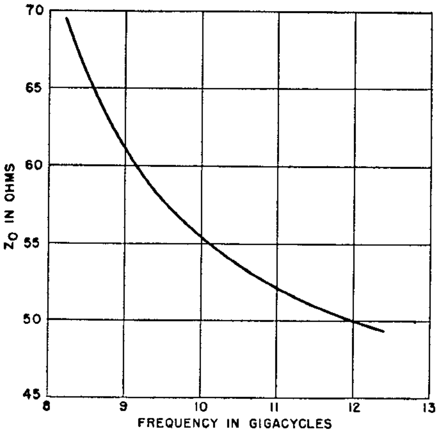

Fig. 5—Characteristic impedance vs frequency for 0.050×0.900 in rectangular waveguide.

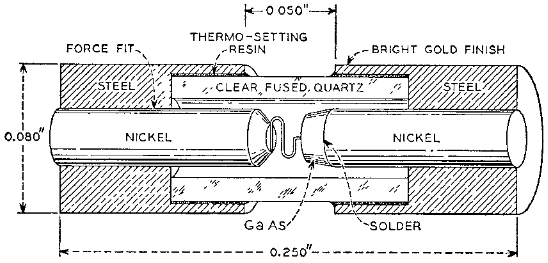

Fig. 6—Sharpless diode cartridge.

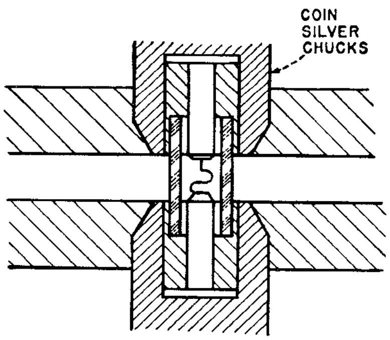

Fig. 7—Cartridge mounting assembly for reduced height waveguide.

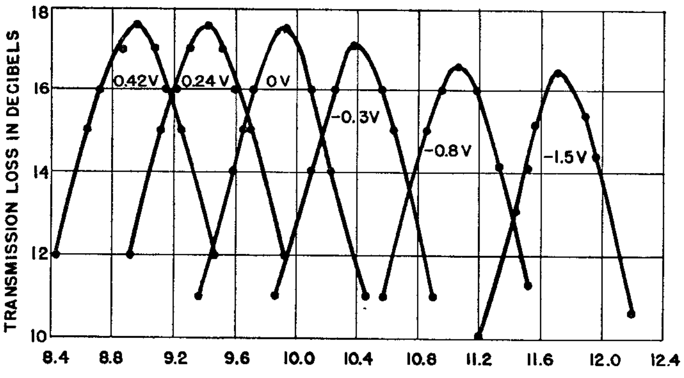

Fig. 8—Transmission loss vs frequency for several different bias voltages for a cartridge mounted diode in reduced height X-band waveguide.

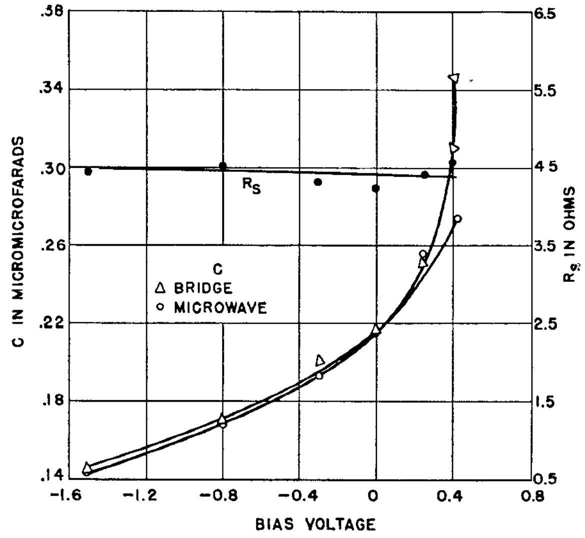

Fig. 9—R_s and C (both microwave and 100-kc bridge measurements.) as functions of bias voltage.

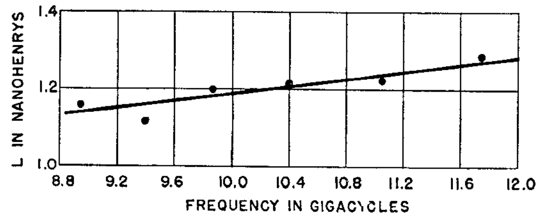

Fig. 10—The effective series inductance of the diode as a function of frequency.

obtained for the corresponding bias voltages at 100 kc on a standard capacitance bridge. The agreement between these two capacitance curves is quite remarkable except in the forward bias region. This difference is real and is substantiated by measurements on many other diodes. Apparently the diffusion capacity (attributed to current flow in the forward bias region) has a frequency dependance rendering it insignificant at X-band frequencies, at least for the diodes measured. This can be seen in Fig. 9 by the continued following of the C-V law of the junction into the forward bias region for the X-band measurements. A dc current of 2.7 μa was associated with the 0.42-v forward bias point, and 0.2 μa at the 0.30-

v forward bias point. R_s is seen to be approximately constant with a slight increase with increasing negative bias (which also unavoidably corresponds to a higher frequency of measurement). Values of R_s so obtained have been observed to differ from those obtained from forward bias slopes (obtained at dc) by as much as 2 to 1. In Fig. 10, L is seen to be a slight function of frequency, but the variation is small enough that our series equivalent circuit representation is an excellent approximation.

It should be remarked that other types of varactor diodes show a marked change of capacity with frequency in the low-frequency range up to a few hundred kc. This has been attributed to surface effects.[7] A similar frequency dependence of R_s has been observed,[8-10] and it is to be expected that in general one must obtain measurements at the frequencies intended for use of the diode in order for them to be meaningful.

VI. Experimental Measurements of Varactor Diodes at *M* Band

These experiments were performed in reduced height (0.148 in × 0.010 in) waveguide. A backward wave oscillator[11] was available which could be swept from 50 to 60 Gc and our measurements are limited to this range. Electroformed raised cosine tapers were again employed in an experimental setup similar to that of Fig. 4 except that in this case no ratio detector was employed. In order to increase our single detection measuring range, backward diodes[12] were used as detectors. The diodes to be measured in this frequency band were built into the waveguide. (Precise cartridge fabrication for 0.010 in height waveguide presents formidable problems.) Thin crystals (0.003 in) of 0.002 to 0.003-Ω cm GaAs were mounted on the ends of 0.032 in rods and pressed in until about 0.002 in of the crystal was protruding into the waveguide. Zinc springs soldered onto similar rods were pressed into the opposite side of the waveguide until contact was made. After electrical forming the transmission curves of Fig. 11 were obtained. The corresponding values of L, C, and R_s are presented in Table I. Although the measuring accuracy is not as good as that at X-band, it is seen from Table I that fairly precise measurements can be obtained even at these frequencies. The reverse breakdown voltage of these diodes varies from 6 to 8 volts. The increase in the effective series resistance at the 0.65 forward bias voltage is due to the fact that we no longer have $R \gg 1/\omega c$. It is to be noted that the cutoff frequency of the diode described in Table I is approximately 800 Gc at zero volts bias. This is to our knowledge the highest cutoff frequency varactor ever reported. This, in conjunction with the abrupt junction C-V curve obtained from the Zn-GaAs system, should make this diode useful in parametric devices well into the millimeter region.

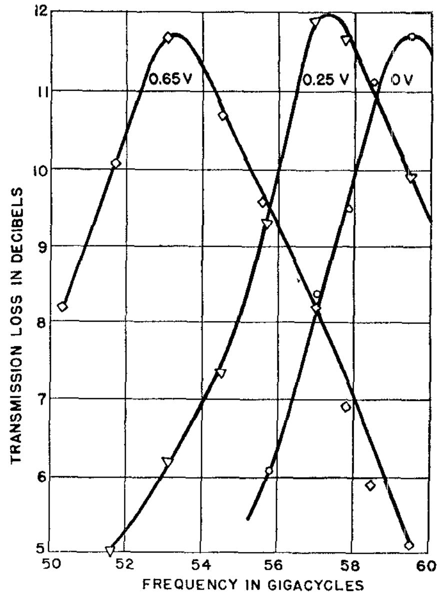

Fig. 11—Transmission loss vs frequency for several different bias voltages for a varactor diode mounted directly in reduced height M-band waveguide.

TABLE I

Bias Voltage	R_s (ohms)	C (picofarads)	L (nanohenries)
+0.65	13.5	0.022	0.40
+0.25	12.1	0.018	0.41
0.00	12.0	0.016–0.017*	0.41–0.42*

* Estimated due to our inability to obtain the upper frequency f_1 which lies above the range of our backward wave oscillator.

VII. Tunnel and Backward Diode Measurements

The techniques of Sections V and VI may be applied to tunnel diode measurements as discussed in Section II. However these measurements are more difficult than varactor measurements because: first, there are only two capacitance values (peak and valley) suitable for measurement, which considerably restricts the range of diodes that can be resonated; second, if the diode is biased so that it cannot switch, one has considerable difficulty in assuring himself that the bias voltage is exactly that of the peak or valley, and is not in effect obtaining some cancellation of the effective series resistance by the diodes negative R. Due to this difficulty we prefer the following approach. The diode is biased so that it can switch. A series of measurements similar to those of Figs. 8 and 11 is begun at voltages just to the right of the valley voltage at point D or

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[7] H. E. Elder, private communication.

[8] D. E. Sawyer "Surface-dependent losses in variable reactance diodes," *J. Appl. Phys.*, vol. 30, pp. 1689–1891; November, 1959.

[9] S. T. Eng, "Characterization of microwave variable capacitance amplifiers," IRE Trans. on Microwave Theory and Techniques, vol. 9, pp. 11–21; January, 1961.

[10] L. D. Braun, "Frequency dependence of the equivalent series resistance of varactor diodes and its effect on parametric amplification," Proc. IRE (*Correspondence*), vol. 50, pp. 2523–2524; December, 1962.

[11] Supplied by Dept. 2813 of Bell Telephone Labs., Murray Hill, N. J.

[12] Supplied by C. A. Burrus, Bell Telephone Labs., Holmdel, N. J.

(for peak L, C, and R_s measurements) just to the left of point C in Fig. 2. Measurements are continued toward the peak or valley voltages until a point is reached at which the microwave measuring power causes the diode to switch. R_s approaches a limiting value under these conditions which should be very close to the actual value, provided low power measurements are made at relatively high microwave frequencies (so that in order to obtain $R \gg 1/\omega c$ one does not have to approach the valley voltage too closely).

Due to the relatively broad range over which backward diodes can have values of R which are large compared to the junction capacitance reactance, they are in general somewhat easier to measure than tunnel diodes. We have, to date, utilized the transmission technique to measure tunnel diodes at X band and backward diodes in the 50–60 Gc range.[12]

VIII. Discussion and Conclusions

A series of experiments has been utilized to provide considerable insight into effective equivalent circuits for microwave diodes in reduced height waveguide. This in turn has provided a simple technique for accurately measuring diode junction properties at high microwave frequencies. Indirectly this has shown that for some varactor diodes (see Section V) the effective microwave junction capacitances can be measured quite accurately on an ordinary low-frequency bridge, as long as one is in the bias region where $R \gg 1/\omega c$.

The frequency sensitivity of the diffusion capacity noted in Section V indicates that the effective capacities of other types of diodes, tunnel diodes for instance, which are usually measured at bias voltages at which dc current is flowing, should be measured at the frequency for which operation is intended. The measurements can of course be made at a lower frequency, provided that it can be established that the frequency sensitivity of the capacitance has disappeared.

The principal limitation of this measuring technique is that of range of measurement. By changing the dimensions of the contacting element one can in practice readily obtain variations of approximately 2 to 1 in the magnitude of the series inductance. For a fixed inductance one can resonate a range of capacitance of approximately 2 to 1 over the nominal operating range of X-band waveguide. These two variations in combination allow approximately a 4 to 1 variation in capacitance measuring range. With varactors, a further extension of range is obtained through the variation of junction capacity with bias. With other types of diodes, such as tunnel or backward diodes, a change in the height of the waveguide and/or the measuring frequency may be necessary in order to accommodate extensive capacity variations. This problem can be somewhat alleviated at low microwave frequencies[3] by the inclusion of a stub turner but this introduces other difficulties.

The 800-Gc cutoff frequency varactor described in Section VI is to our knowledge the highest cutoff frequency diode ever reported. Its use should extend the use of parametric devices well into the millimeter region. The transmission technique herein described is to our knowledge the only technique available for accurately measuring the properties of this diode in the 50–60 Gc frequency range.

Appendix I

For $T \leq 2$, C as determined from (2) is either zero or complex due to the $\sqrt{1-2/T}$ factor. This limitation is due to our basing (2) on the double power frequencies which do not exist for $T \leq 2$. Actually as T approaches 2 (with T greater than 2), f_1–f_2 becomes infinite. Thus by our choice of f_1 and f_2 we will be limited to values of R_s such that

$$R_s < 1.22 Z_0. \tag{5}$$

If one has $R_s > 1.22\ Z_0$, he can still utilize this technique but must select f_1 and f_2 such that the power transmitted to the load of these frequencies is less than twice that transmitted at resonance. Eqs. (2) and (3) must than of course be suitably modified.

Appendix II

Eqs. (2) and (3) are derived on the assumption that the transmission line impedance at frequencies f_1 and f_2 is the same as the transmission line impedance at resonance. This is not true in a dispersive medium and in cases where this is not a good approximation one must use the following exact expressions.

$$C = \frac{1}{2\pi f_1 f_2}\left[\frac{f_1^2 - f_2^2}{f_2\delta_1 + f_1\delta_2}\right],$$

$$L = \frac{1}{2\pi}\left[\frac{f_1\delta_1 + f_2\delta_2}{f_1^2 - f_2^2}\right],$$

where

$$\delta_1 = \left\{\frac{2Z_1\{Z_0 + Z_1(\sqrt{T} - 1)\}^2 - Z_0^3 T}{4(Z_0 T - 2Z_1)(\sqrt{T} - 1)^2}\right\}^{1/2}$$

and where

$$\delta_2 = \left\{\frac{2Z_2\{Z_0 + Z_2(\sqrt{T} - 1)\}^2 - Z_0^3 T}{4(Z_0 T - 2Z_2)(\sqrt{T} - 1)^2}\right\}^{1/2},$$

with δ_1 and δ_2 both positive, and where Z_1 is the transmission line impedance at f_1, Z_2 is that at f_2 and Z_0 is that at resonance.

Acknowledgment

The author is pleased to acknowledge the assistance of S. F. Jankowski who made the X-band measurements and assisted in the computations. The GaAs utilized in the M-band measurements was supplied by the Bell Telephone Laboratories group at Allentown, Pa. The author wishes in particular to acknowledge the cooperation of N. C. Vanderwal of that group.